

We recall that $G \rightarrow H$ denotes that there exists a homomorphism from G to H .

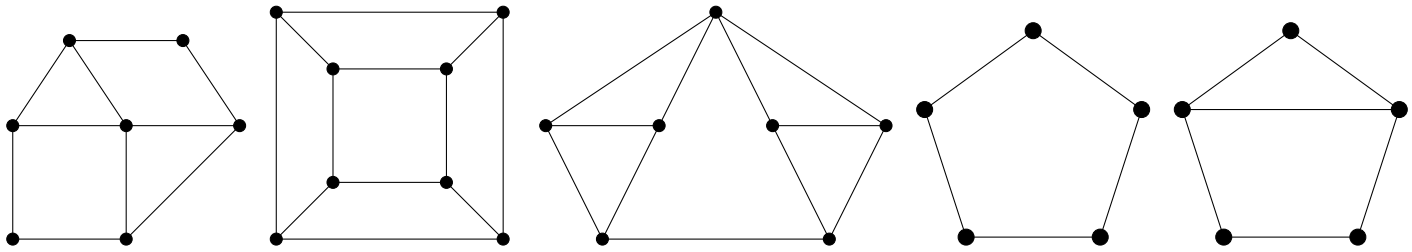
Exercise 1

Find a graph with as few vertices as possible that does not have a homomorphism to the cycle C_6 on six vertices. Justify your answer.

Exercise 2

Among the following graphs, find which ones are homomorphic to which other ones.

Find the cores of each of the graphs. Justify your answers.



Exercise 3

Build two graphs G and H such that there is no homomorphism from G to H nor from H to G .

Exercise 4

Build a graph H such that for every planar graph G with at least one edge, $H \rightarrow G$.

Build a graph H such that for every planar graph G with at least one edge, $G \rightarrow H$. Justify your answers.

Exercise 5

Show that if $G \rightarrow H$, then :

- $\omega(G) \leq \omega(H)$
- $\chi(G) \leq \chi(H)$

Exercise 6

Show that a graph G is a core if and only if every endomorphism of G is an automorphism.

Exercise 7

This exercise is a bit more open, and you are not expected to have a complete answer.

The square of a cycle on k vertices, denoted C_k^2 , is defined as the graph with vertices $v_1, v_2 \dots v_k$ so that $v_i v_j \in E(G)$ if and only if $|i - j| = 1[k]$ or $|i - j| = 2[k]$. That is, C_k^2 is exactly the cycle on k vertices where edges are added between vertices that share a neighbour.

For which values of i and j can you say that $C_i^2 \rightarrow C_j^2$? For which values can you say that this does not hold.

Justify your answer. (hint : you may want to consider the values of k modulo 3).